Aufgabe 1

Mit der Formel

$m = 2^{12} = 4096; \quad A = \frac{\pi-2}{3}$

a)

$$k = 1000$$

$$\begin{aligned} h &= \left(1000 * \frac{\pi-2}{3} - \lfloor 1000 * \frac{\pi-2}{3} \rfloor\right) * 4096 \\ &= 2174 \end{aligned}$$

b)

$$k = 2500$$

$$\begin{aligned} h &= \left(2500 * \frac{\pi-2}{3} - \lfloor 2500 * \frac{\pi-2}{3} \rfloor\right) * 4096 \\ &= 1340 \end{aligned}$$

Mit Ganzzahlarithmetik

Approximation

$$\begin{aligned} &\left|\frac{S}{2^{16}} - A\right| && \text{minimal} \\ &\frac{S}{2^{16}} - A = 0 \\ &S = \lceil A * 2^{16} \rceil = 24939 \end{aligned}$$

a)

$$k = 1000$$

$$\begin{aligned} r &= s * k \mod 2^{16} \\ &= 2493900 \mod 65536 \\ &= 35320 \end{aligned}$$

$$\Rightarrow \underbrace{1000\,1001\,1111}\,1000$$

$$h(1000) = 2207 = \lfloor \frac{35320}{2^4} \rfloor$$

b)

$$k = 2500$$

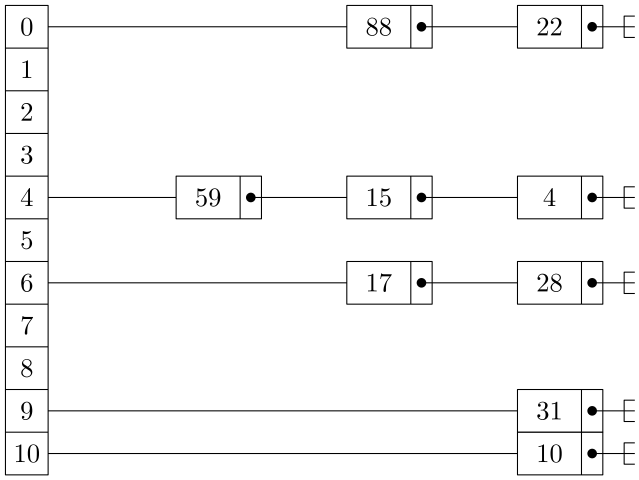
$$\begin{aligned} r &= s * k \mod 2^{16} \\ &= 62347500 \mod 65536 \\ &= 22764 \end{aligned}$$

$$\Rightarrow \underbrace{0101\,1000\,1110}\,1100$$

$$H(2500) = 1422 = \lfloor \frac{22764}{2^4} \rfloor$$

Aufgabe 2

a)



b)

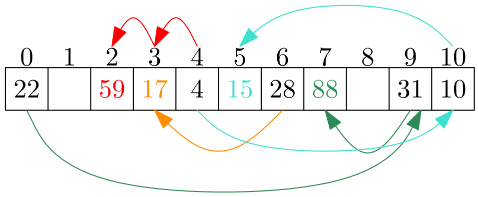
0	1	2	3	4	5	6	7	8	9	10
22	88			4	15	28	17	59	31	10

c)

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
15	17			4		22		88		10	59	28			31

$$h(k,i) = (k + \frac{1}{2}i + \frac{1}{2}i^2) \mod 16$$

d)



$$
\begin{aligned}
h_1(k) &= k \mod 11\\
h_2(k) &= 1 + (k \mod 10)\\
h(k,i) &= \Big(h_1(k) + i * h_2(k)\Big) \mod 11
\end{aligned}
$$

Sondierungsfrequenz für k=31

i	0	1	2	3	4	5	6	7	8	9	10
$h(k,i)$	9	0	2	4	6	8	10	1	3	5	7

$< 9, 0, 2, 4, 6, 8, 10, 1, 3, 5, 7 >$

Aufgabe 3

Für alle $0 \leq j < m$ gilt:

$$\frac{n-j}{m-j} \leq \frac{n}{m}$$

Annahme: Es gibt ein j mit:

$$
\begin{aligned}
\frac{n-j}{m-j} &> \frac{n}{m} && \mid *(m-j)*m\\
(n-j)*m &> n*(m-j)\\
\cancel{n*m} - j*m &> \cancel{n*m} - j*m && \mid * \frac{1}{-j}\\
m &< n && \lightning
\end{aligned}
$$